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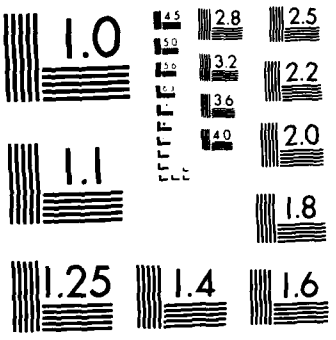

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THE ROLE OF ELLIPTICITY AND NORMALITY ASSUMPTIONS IN FORMULATING LIVE-BOUNDARY CONDITIONS IN ELASTICITY

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ABSTRACT

It is shown that those interactions of an elastic body $\mathcal{B}$ with an elastic environment $\mathcal{E}$ manifested by local surfacial loadings should be modelled by boundary operators featuring a live system of surfacial forces, i.e., a vector field s, defined over the boundary $\partial\mathcal{B}$ of $\mathcal{B}$ and representing the surfacial force per unit area exerted by $\mathcal{E}$ over $\mathcal{B}$, which depends functionally in a non-trivial way on the deformation u of $\mathcal{B}$.

In particular, under the assumptions that the constitutive equation of $\mathcal{B}$ is compatible with ellipticity, and s is a function of the appropriate restrictions to $\partial\mathcal{B}$ of u and its gradient, it is also shown that it is reasonable to require that the resulting live-boundary condition of traction be genuine, i.e., a normality condition prevail insuring the primary consistency of the pair of field and boundary operators.

In addition to the analytical difficulties to be expected (cf. [7], Chapter 2), when normality does not hold the mechanical interpretation of boundary conditions fails to be unique. That this is indeed the case is demonstrated by producing an explicit example in linearized elastostatics of a boundary operator which can be interpreted as a non-genuine live-boundary condition of traction or as a dead-boundary condition of frictionless contact.

AMS(MOS) Subject Classification: 73C50, 35J60.

Key Words: ellipticity, normality, live-boundary condition of traction.

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SIGNIFICANCE AND EXPLANATION

In the real world, most loadings "follow" the deformation or, in the terminology used in this paper, are live, in that their direction and/or their magnitude may vary with the deformation itself (the hydrostatic pressure on a submerged object and jet loading are common examples of live loadings; buckling of elastic rods and plates and panel flutter are examples of statical and dynamical problems from structural engineering and aeroelasticity, respectively, where live loadings make an essential feature).

In spite of its practical importance and deep mathematical interest, a general treatment of live loadings in linear and non-linear (and, especially, three-dimensional) elasticity is still lacking.

The purpose of this paper is to delineate circumstances under which one can expect a given boundary condition to represent realistically an unambiguous mathematical model for live surface loading. It is shown that the ellipticity up to the boundary of the field operator, as well as the normality of the pair of field and boundary operators, both play a crucial role (ellipticity is a requirement of physical and mathematical plausibility of general use in elasticity; normality is a basic mutual consistency requirement for field and boundary operators in the theory of elliptic systems, receiving a precise mechanical interpretation in this paper).

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THE ROLE OF ELLIPTICITY AND NORMALITY ASSUMPTIONS IN FORMULATING LIVE-BOUNDARY CONDITIONS IN ELASTICITY

P. Podio-Guidugli*, G. Vergara Caffarelli** and E. G. Virga***

1. Introduction

The mechanical interaction of a body with its environment is a varied and complex phenomenon which is difficult both to observe and describe carefully. Boundary conditions modelling non-trivial interactions are difficult to formulate mathematically; besides, they may easily lead to unusual boundary value problems, perhaps even so much unusual as to challenge and defeat an expert analyst. Not only that, blunt generalization of the few well-understood types of boundary conditions may pose rather subtle problems of interpretation and classification to the expert mechanist.

In Section 2, the bulk of this paper, we develop a train of reasoning leading to formulate an unambiguous boundary condition of traction, reproducing the non-linear interaction of an elastic body and an elastic environment in strict contact with it; the emphasis here, in accordance with our title, is on the role played by ellipticity and normality hypotheses.

In Section 3, we briefly discuss the importance of ellipticity and normality in problems of elastostatics linearized about a stressed equilibrium placement. In particular, we show by means of an explict example that, if the environment is live but normality fails to hold, one can easily run into taxonomic troubles.

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2. The Genuine Live-Boundary Condition of Traction

Let a (possibly unbounded) open regular region $\mathcal{B}$ of $\mathbf{R}^M$, with $M = 2$ or 3, be separated by its exterior $\mathcal{E}$ by a smooth orientable surface $\mathcal{S}$, the common boundary of $\mathcal{B}$ and $\mathcal{E}$, with $\mathbf{n}$ the exterior unit normal to $\mathcal{B}$.

Once constitutive prescriptions have been given, we may think of $\mathcal{B}$ as the body, and of $\mathcal{E}$ as the environment of the body $\mathcal{B}$.* Then, given any judiciously chosen set of data, we may formulate a variety of mechanically significant problems, anyone of which ultimately reduces to determine the state (i.e., the triplet of displacement, strain and stress) over $\mathcal{B} \cup \mathcal{E}$, provided a suitable set of jump conditions has been stipulated for the state at the points of $\mathcal{S}$.

Loosely, our goal is to sketch a credible scenario where a large class of such problems for $\mathcal{B} \cup \mathcal{E}$ can be reduced through a rational procedure to one corresponding problem for $\bar{\mathcal{B}} = \mathcal{B} \cup \mathcal{S}$, with the presence of $\mathcal{E}$ replaced by a set of boundary conditions on $\mathcal{S}$. To anticipate one of our main findings, perhaps not about to come as any great surprise to the knowledgeable reader, these boundary conditions will as a rule be of the live type, i.e., they will be prescriptions of the surfacial stress vector which depend functionally in a non-trivial way on the solution in $\bar{\mathcal{B}}$.

Now, let $\mathbf{u}$, $\mathbf{T}$ denote the restrictions to $\mathcal{S}$ of the unique extensions $\dot{\mathbf{u}}$, $\dot{\mathbf{T}}$ to $\bar{\mathcal{B}}$ of smooth displacement and stress fields $\mathring{\mathbf{u}}$, $\mathring{\mathbf{T}}$ over $\mathcal{B}$; further, let $(\mathbf{u},\mathbf{t})$, with $\mathbf{t} = \mathbf{T}\mathbf{n}$, denote the pair of fields over $\mathcal{S}$ consisting of the displacement $\mathbf{u}$ and the stress vector $\mathbf{t}$; finally, let $(\mathbf{v},\mathbf{s})$

* Notice that here we implicitly choose to not consider the interesting case when $\mathcal{S}$ itself is thought of as a material surface (in the manner, e.g., of GURTIN & MURDOCH [1]), and, therefore, made the object of a peculiar constitutive prescription. To contemplate such a case would require a more delicate analysis.

be defined over S in a completely analogous way, but starting from displacement and stress field $\overset{\circ}{v}$ and $\overset{\circ}{S}$ defined over E. We wish to interpret $v(x)$, $s(x)$ as, respectively, the displacement of boundary point x of the environment and the contact force exerted at x by the environment over the body.

A rather spontaneous assumption is that such contact force depends functionally on the displacement field over $E \cup S$. Suppose that the stress response of E to deformation have a local character, as is the case for an elastic environment. Then, for $x \in S$ fixed, and for any given field $\overset{\circ}{v}$ on E, one finds natural to choose the functional dependence of s on $\overset{\circ}{v}$ as follows

$$s(x) = \hat{f}(v(x), D\overset{\circ}{v}(x)), \tag{2.2}$$

where D is the gradient operator.

A slight generalization of such a choice, of importance in our further developments, is suggested by the splitting of the gradient operator D, at a point $x \in S$, into its normal and tangential parts D_n and D_t:

$$D\overset{\circ}{v}(x) = D_n\overset{\circ}{v}(x) + D_t\overset{\circ}{v}(x) , \tag{2.3}$$

with

$$D_n\overset{\circ}{v}(x) = \partial_n\overset{\circ}{v}(x) \otimes n(x) , \quad D_t\overset{\circ}{v}(x) = D_t v(x) , \tag{2.4}$$

and with ∂_n denoting the operation of differentiating along the direction $n(x)$, the normal to S at x. Then, in view of (2), (3) and (4), one is induced to replace (2) by

$$s(x) = \hat{f}(v(x), D_t v(x), \partial_n\overset{\circ}{v}(x)) . \tag{2.5}$$

Our next step is to lay down a set of jump conditions for the state at the points of S.

We say that the body B and its environment E have a strict contact in a neighbourhood of a point x of their surface of separation S if

$$u(x) = v(x) \quad \& \quad t(x) = s(x) \,. \tag{2.6}$$

In a strict contact, both the displacement and its tangential gradient suffer no jump at S; besides, the continuity of the stress vector at S implies in general restrictions on the jump condition for the normal derivative of displacement. To motivate the ad hoc assumptions we shall make to determine those restrictions, we turn momentarily to linear elasticity.

Suppose that B and E, besides being in strict contact, are made up with two linearly elastic materials with elliptic elasticity tensors $\mathbf{C}$ and $\mathbf{D}$, respectively, so that $\mathbf{C}$, say, obeys

$$(\underline{\mathrm{EC}})^{\circ} \qquad \det \mathbf{C}^{\circ}(a) \neq 0 \quad \text{for} \quad \mathbf{C}^{\circ}_{ih}(a) = \mathbf{C}_{ijhk} a_j a_k, \; a \neq 0.$$

If $\mathbf{C}$ ($\mathbf{D}$) can be firstly extended and then restricted to S in such a way as to preserve ellipticity, the matrix field

$$A = \mathbf{C}^{\circ}(n) \quad (B = \mathbf{D}^{\circ}(n)) \tag{2.7}$$

has invertible values on S. Thus, as

$$t = (\mathbf{C}\dot{\mathbf{D}}u)n = A\partial_n \dot{u} + (\mathbf{C}\mathbf{D}_t u)n, \tag{2.8}$$

we have that

$$\partial_n \dot{u} = A^{-1}(t - (\mathbf{C}\mathbf{D}_t u)n) \,, \tag{2.9}$$

a formula which shows that, once u is assigned on S, t and $\partial_n \dot{u}$ are essentially interchangeable bits of information at a point of S. Likewise,

$$\partial_n \dot{v} = B^{-1}(s - (\mathbf{D}D_t u)n) \ . \tag{2.10}$$

Thus, in particular, if $\mathcal{B}$ and $\mathcal{E}$ are in strict contact, linearly elastic and elliptic, the normal derivative of displacement suffers at S a discontinuity depending on the local values of the stress vector and the tangential gradient of displacement, as **(A,C)** and **(B,D)** may be expected, in general, to differ.

Remark 1 It is important to note that ellipticity, and not linearity of the constitutive law, is the crucial assumption. Indeed, for $\mathcal{B}$ comprised of a non-linearly elastic material, $(8)_2$ is replaced by

$$t = T(\partial_n \dot{u}\otimes n + D_t u)n \ , \tag{2.11}$$

and this equation is locally solvable for $\partial_n \dot{u}$ if the Ellipticity Condition prevails, i.e.,

$$(\underline{\mathrm{EC}}) \qquad \det C(a) \neq 0 \quad \text{for} \quad C_{ih}(a) = \mathbf{T}_{ijhk} a_j a_k \ , \quad a \neq 0, \quad \mathbf{T} = \frac{\partial T}{\partial (D\dot{u})} \ .$$

Remark 2 For boundary-value problems ruled by PDE's, ellipticity hypotheses typically involve the coefficients of the field operator; here, EC plays an important role at the boundary.

In the light of the above, if one assumes that $\mathcal{B}$ and $\mathcal{E}$ have a strict contact, then (6) allows one to give (5) the following provisional form:

$$s(x) = \hat{s}\big(u(x), D_t u(x), \partial_n \dot{v}(x)\big) \ . \tag{2.12}$$

Moreover, for elliptic elastic pairs of a body $\mathcal{B}$ and an environment $\mathcal{E}$, it seems safe to assume that both $\partial_n \dot{u}$ and $\partial_n \dot{v}$ depend on the tangential gradient and the stress vector

$$\partial_n \dot{u} = u(D_t u, t), \quad \partial_n \dot{v} = v(D_t u, s) \tag{2.13}$$

and that the dependence on the stress vector is essential, in that the matrices $\frac{\partial u}{\partial t}$, $\frac{\partial v}{\partial s}$ are invertible.

By (12) and (13), there exists a function g such that

$$s(x) = g\big(u(x),\ D_t u(x),\ s(x)\big)\ . \tag{2.14}$$

Last step towards our goal, we assume the condition (14) is genuine, in that for no assignment of u, $D_t u$ it does reduce to an identity in s. Accordingly, we require that

$$\det L \neq 0, \quad \text{with} \quad L = 1 - \frac{\partial g}{\partial s}\ . \tag{GC}$$

GC implies that (14) is locally solvable for s; then, s can be expressed as a function of u and $D_t u$:

$$s(x) = \delta\big(u(x),\ D_t u(x)\big)\ . \tag{2.15}$$

The last formula suggests that in formulating boundary value problems the contact force exerted by the environment on the body ought chosen in general to be of the live type.

Remark 3 The special dependence (15) of s on boundary information concerning the solution has been considered by SPECTOR [2,3], who has termed simple such surfacial loading operators. SPECTOR has motivated his choice by the observation, due to GURTIN [4], that pressure loading, a well-known example of live loading, is indeed simple.

Remark 4 For two linearly elastic elliptic bodies in strict contact, (12) takes the form

$$s = B\partial_n \dot{v} + (\mathbf{D}D_t u)n$$

and is readilly inverted as

$$\partial_n \dot{v} = B^{-1}(s - (\mathbf{D}D_t u)n) \ , \tag{2.10}$$

which in turn corresponds to $(13)_2$. Therefore, (14) reduces to a sheer identity, and GC cannot be satisfied. This indicates that, in general, it is not sound to require GC in a contact situation. However, on passing from contact to boundary conditions, we believe that one can regard $(13)_2$ as a separate constitutive assumption, independent of (12) and consistent with it in the peculiar sense made explicit by GC.

Substitution of (15) into $(13)_2$ and, when use is made of $(6)_2$, into $(13)_1$, allows us to complete with a suitable jump condition for the normal derivative of displacement the set of jump conditions for the state to be stipulated at a point of S. In summary, under hypotheses of strict contact, ellipticity and genuinity, the displacement u and its tangential gradient $D_t u$ must be continuous, whereas the normal derivative $\partial_n \dot{u}$ of displacement may suffer a jump described by a function of u and $D_t u$, whose choice has constitutive character.

We are now in a position to state a boundary condition of traction which would describe a large class of contact actions exerted by an environment of elastic type on an elastic body.

Naively, one would write a live-boundary condition of traction for $\mathcal{B}$ as

$$t(x) = s(x) \quad \text{for} \quad x \in S,$$

or rather (cf. (2) and (11)), as

$$\bar{\ell}\,(u,\, D_t u,\, \partial_n \dot{u})(x) = 0 \quad \text{for} \quad x \in S, \tag{2.16}$$

with the boundary operator $\bar{\ell}$ defined by

$$\bar{\ell}(u,\, D_t u,\, \partial_n \dot{u}) = T(\partial_n \dot{u} \otimes n + D_t u)n - \hat{\mathcal{f}}(u,\, \partial_n \dot{u} \otimes n + D_t u)\,. \tag{2.17}$$

On the other hand, in view of (6), $(13)_1$ and (15), the boundary operator should have the following form

$$\ell(u,\, D_t u,\, \partial_n \dot{u}) = \partial_n \dot{u} - \mu(u,\, D_t u)\,. \tag{2.18}$$

Now, $\bar{\ell}$ is locally reducible to ℓ if the implicit equation

$$\bar{\ell}(u,\, D_t u,\, \partial_n \dot{u}) = 0$$

can be locally solved for $\partial_n \dot{u}$; this is the case if the following Normality Condition holds:

$$(\underline{\text{NC}}) \qquad \det M(n) \neq 0 \quad \text{for} \quad M(n) = C(n) - \mathbf{f}n, \quad \mathbf{f} = \frac{\partial \hat{\mathcal{f}}}{\partial(D\dot{u})}\,.$$

Within the scenario we have set up, accepting NC is mandatory. We observe that EC implies NC when the environment is dead, and $\hat{\mathcal{f}}$ reduces to an assigned function of x only over ; more generally, when $\hat{\mathcal{f}}$ is independent of the deformation gradient; finally, when $\hat{\mathcal{f}}$ is simple in the sense of Remark 3.

We call (16), with $\bar{\ell}$ restricted by NC, the genuine live-boundary condition of traction.

3. Ellipticity and Normality in Linearized Elastostatics

When one passes from a local study of a non-linear problem to the corresponding linearized problem, both EC and NC continue to play an important role. Indeed, our present NC is a version appropriate to elasticity of the homonymous condition introduced by GEYMONAT [5] and GRUBB [6] in the general theory of linear elliptic systems, and appears to be the natural extension to systems of the normality condition for scalar operators (vid. e.g. [7], Chapter 2).

If EC and NC prevail, we have shown in [8] that a Green formula, which generalizes the reciprocity formula of Betti of classical elasticity, can be associated to the traction boundary value problem of linearized elastostatics with live loads. Such a Green formula allows one: (i) to define formal adjoints to both the field and boundary operator; (ii) to determine a set of conditions sufficient for self-adjointness; (iii) to state compatibility conditions on the data necessary and sufficient [9] for solvability of the underlying boundary-value problem in a familiar Hilbert space setting.

We now wish to give an example of the ambiguities that may accompany the failure of NC.

In view of (2.16), a linearized live-boundary condition of traction can be written as

$$(TD\dot{u}(x))n(x) - \mathbf{f}D\dot{u}(x) - Fu(x) = 0, \tag{3.1}$$

where u, with slight abuse of notation, is now used for the first approximation of the displacement from the reference placement;

$$t(x) = (TD\dot{u}(x))n(x); \tag{3.2}$$

$$s(x) = \mathbf{f}D\dot{u}(x) + Fu(x), \quad \text{with} \quad F = \frac{\partial \hat{f}}{\partial u}. \tag{3.3}$$

Choose $\hat{\mathcal{S}}$ to describe a live environment such that

$$f_{ihk} = P_{il}\mathbf{T}_{ljhk}n_j, \quad F_{ih} = -P_{ih}\,, \tag{3.4}$$

with P a perpendicular projection field over S. Then, it is easy to check that the matrix $M(n)$, which appears in the statement of NC, reduces to

$$M(n) = (1 - P)C(n), \tag{3.5}$$

and is, therefore, singular, whereas (1) takes the aspect

$$(1 - P)t + Pu = 0, \tag{3.6}$$

or, equivalently,

$$(1 - P)t = 0 \quad \& \quad Pu = 0\,. \tag{3.7}$$

One promptly recognizes in (7) a generalized boundary condition of the dead type (cf. [10], Chapter V). In particular, when $P = n\otimes n$, (7) reduces to the so-called contact boundary condition:

$$t - (t\cdot n)n = 0 \quad \& \quad u\cdot n = 0\,. \tag{3.8}$$

Thus, if f, F as in (4) and $P = n\otimes n$, the boundary condition (1) can be quite well classified as a live-boundary condition of traction or as a dead-boundary condition of contact.

Other similar examples can be concocted. But, the validity of NC precludes any pathology.

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